Chapter 6

Shear Strength of Soil

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Chapter 6
SHEAR STRENGTH OF SOIL

6.1 Introduction

If the load or stress in a foundation or earth slope is increased to cause an unacceptably large deformation, the soil in the foundation or slope is considered as failed against its strength. Geotechnical engineers always consider strength of the soil as the shear strength. This is because the tensile strength of soil is very small and practically accepted to be zero. The applied compressive load eventually causes the soil to fail in shear in a rupture surface. As the soil is a granular or particulate material, the excessive deformation, as mentioned, generally occurs due to sliding, rolling or rearrangements in some way of the particles within a soil mass or simply due to shearing. This phenomenon is illustrated in Fig. 6.1.

![Illustration of soil failing in shear due to sliding and rolling of particles past each other.](image)

Therefore, the sliding capability of a soil, while to support a loading from a structure or to support its own overburden or to sustain a slope in equilibrium, is called shear strength of soil. As it is referred to the strength, it is the maximum or ultimate shear stress that the soil can sustain to continuous shear displacement of soil particles along a surface of rupture.

6.2 Importance of Shear Strength in Soil Mechanics

In many of the soil mechanics problems, the shear strength of the soil emerges as one of the most important characteristics. The examples are

- Foundation design
- Earth and rockfill dam design
- Highway and airfield design
- Stability of slopes and cuts
- Lateral earth pressure
6.3 Contributing Factors of Shear Strength of Soil

The shear strength of the soil may be attributed to three basic components:

- Resistance due to interlocking of the particles
- Frictional resistance between the individual soil grains, which may be either of the sliding or rolling frictions or the both.
- Adhesion between soil particles called cohesion

*First two sources are mainly responsible for the shear strength of granular soils; second and third for the cohesive soils. Whereas, third one is only responsible for shear strength of highly plastic clays. It is neither easy nor practical to clearly distinguish the effects of these components on the shear strength of the soil, because these components in turn are influenced by many factors like:*

(i) Heterogeneous nature of soil that typify most soil masses  
(ii) The water table location and moisture contents  
(iii) The drainage facility involving pore water pressure  
(iv) The type and nature of construction  
(v) Stress history  
(vi) Structural disturbance of soil  
(vii) Chemical action  
(viii) Environmental conditions

Therefore, the shear strength of a soil cannot be interpreted in a very simple way since a soil can significantly exhibit different shear strength at various field conditions. The shear strength (or its parameters) and deformation of a foundation can be obtained either from laboratory tests on carefully extracted field samples or from in situ tests. However, *the understanding of these interrelated and complex matters needed to have a clear in view of the following basic aspects.*

- Friction between solid bodies  
- Cohesion between the soil particles  
- Stresses on the planes passing through a point represented by Mohr Circle diagram

6.3.1 Friction between solid bodies

This is similar to classic sliding friction problem from basic physics or mechanics. The force that resists sliding equals to normal force multiplied by a coefficient of friction, \( \mu \).

Let us consider a block of weight, \( W \) placed on a horizontal plane having frictional interface as shown in Fig. 6.2. The forces acting on the block, Fig. 6.2a, are gravity and the reaction of the plane. Although a friction resistance is available, it does not come into play since there is no horizontal force applied to the block.

Now, a horizontal force \( P_i \) is applied to the block, Fig. 6.2b. If \( P_i \) is small, the block will not move; the resultant \( R_a \) makes an angle \( \alpha \) commonly known as angle of obliquity of force \( P_i \). For equilibrium, a resisting horizontal force \( F_i \) must therefore exist to balance \( P_i \). The angle \( \alpha \) is less
than the maximum angle available, for the maximum horizontal force, known as angle of friction, $\phi$. In Figure, $N$ is the component of the weight normal to the plane.

If the applied horizontal force is increased to $P_2$, as shown in Fig. 6.2c, the friction force also increases to a value of $F_2$. It reaches to a maximum value when the angle of obliquity $\alpha$ is equal to the angle of friction, $\phi$. At this point a state of impending motion exists. For the range of value $0 < \alpha < \phi$ no slip or sliding occurs. At this point of impending motion, the maximum available frictional force $F$ can be related to the normal resisting force by a coefficient $\mu$ as given by

$$F = \mu N$$

(6.1)

![Fig. 6.2 Illustration of frictional forces. (a) Development of no friction; (b) Development of friction; (c) Friction impending motion; (d) Friction with motion.](image)

The coefficient of friction $\mu$ is independent of the area of contact. It is, however, strongly dependent on the nature of the surface in contact, the type of material, the condition (wet or dry) of the surface and so on.

If the horizontal force is increased to a value of $P_3$ as shown in Fig. 6.2d, such that the angle $\alpha$ is greater than $\phi$, the block will start sliding. The frictional force cannot exceed the value given by $F = \mu N$, and therefore the block will accelerate in the direction of applied horizontal force. Fig. 6.2c shows that $\mu = \frac{F}{N} = \tan \phi$, which also means that $\sigma \tau = \tan \phi$. 

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6.3.2 Concept of Cohesion

The soil mass consists of granular materials. There exists two forces namely, body force and Van der Wall force, between two bodies which usually govern the the properties of granular mass. The body force is usually referred to as gravitational force where the mass of the bodies are predominant and larger particles are influenced by this force. Whereas, Van der Wall force is due to electrical surface charges and a bonding between the particles occurs in presence of water which helps to generate various chemical bonding among the particles. This adhesion due to bonding is known as cohesion for soil. In case of smaller soil particles the specific surface area (area per unit mass) is significantly higher as compared to the coarse grained soil. As the electrical charges and hence the Van der Wall force are directly proportional to surface charge, the property of fine grained soil is governed by cohesion of the soil. The body force here is insignificant because of their smaller mass weight. Similar thing happens to coarse grained soil; the Van der Wall force here has a very small influence.

However, this cohesion parameter is dependent on stress history that is mode of formation of soil deposit. In soils like overconsolidated clays, partially saturated soils and cemented soils the individual particles are bonded together giving rise to cohesion. As such, this parameter can be viewed as a source of shear strength that is independent of normal force.

6.3.3 Stresses on planes passing through a point and Mohr’s circle

According to principles of mechanics, both normal and shear stresses act on planes passing through a point that has been subjected to external load. These stresses may be represented graphically by an extremely useful device known as Mohr’s circle of stress. The device is based on a unique point on a circle called the pole or origin of planes. This point has such a useful property that “any straight line drawn through this point (pole) will intersect the circle at a point which represents the state of stress on a plane inclined at the same the orientation in the space of the line.” According to this Pole method, at least two lines from known points of stresses on Mohr’s circle parallel to the corresponding planes on which they act are drawn to intersect a point on the circle which is the pole. Once the pole is known the stresses on any plane can be found by drawing a line from the pole parallel to that plane; the coordinates of the point of intersection with the Mohr’s circle determine the stresses on that plane. Mohr’s circle in soil mechanics follows certain sign convention.

- The compressive stress is considered as positive
- The counter-clockwise shear stress is positive

The horizontal and vertical coordinates represent normal and shear stresses respectively. For example, in Fig. 6.3(a) the normal and shear stresses are acting on the planes of a rectangular element. Now, let us consider a plane on the element (Fig. 6.3a) making an angle \( \theta \) with the horizontal. If a line is drawn parallel to this plane from the pole point \( P \) to intersect the circumference of the Mohr’s circle at a point that will represent the stresses \((\sigma', \tau)\) acting on the plane in question.

In most of the soil mechanics problems only the top half of the Mohr’s circle is drawn for convenience. Mohr’s circles are usually drawn for principal stresses and the pole lies on the normal stress \((\sigma')\) axis. The other point is that there would be two failure planes in such a case. This has got a little significance because of the fact that in practical field condition the particulate material soil will initiate its failure at the weakest of the two planes.
Mohr Circle of stress
Soil element

Resolving forces in $\sigma$ and $\tau$ directions,

\[
\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta
\]
\[
\sigma' = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta
\]

\[
\tau^2 + \left(\sigma' - \frac{\sigma_1 + \sigma_3}{2}\right)^2 = \left(\frac{\sigma_1 - \sigma_3}{2}\right)^2
\]

(a)

(b)

P_d = Pole w.r.t. plane

Fig. 6.3 (a) Stresses on an element; (b) Illustration of Pole of Mohr circle.

6.4 Mohr-Coulomb Failure Theory

Mohr (1900) presented a theory for rupture of materials. This states that the failure in materials occurs with a critical unique combination of normal and shear stresses on the rupture surface and not solely by either of the maximum normal or maximum shear stresses. This can be expressed as a function in the form

\[
\tau_f = \oint \mathbf{f} \cdot d\mathbf{s}
\]  

(6.2)
The stress \( \tau \) at failure (rupture) plane is called the shear strength of the material. \( \sigma \) is the normal stress on the failure plane. Fig. 6.4 shows the relationship as expressed by Eq. (6.2) and the element at failure with principal stresses causing the failure.

![Mohr's failure criteria](image.png)

So, if somehow, the principal stresses at failure can be estimated, a Mohr’s circle can be constructed to represent this state of stress. Similarly, several tests to failure at various combination of principal stresses would lead to construct several Mohr’s circles. Such a series is plotted in Fig. 6.5. Since the Mohr’s circles are determined at failure, it is possible to construct limiting or failure envelope of the shear stress. This envelope is called the Mohr’s failure envelope which expresses the relationship between \( \tau_f \) and \( \sigma_f \) as represented by Eq.(6.2) and Fig. 6.4(a). It is to be taken in to note that the failure envelope defined by Mohr (Fig. 6.4a) is a curved line.

It is worth mentioning that Mohr circle lying below the envelope, shown as a dotted circle in Fig. 6.5, represents a stable condition and the circle going above the failure envelope cannot exists. Failure occurs only when the combination of normal and shear stresses is such that the Mohr’s circle is tangent to the Mohr’s failure envelope. If this envelope is unique for a given material, then the point of tangency of the Mohr failure envelope defines the condition on the failure plane at failure. Using the Pole method, the angle of friction plane from the point of tangency of the Mohr circle and the Mohr failure envelope. That is, Mohr’s failure hypothesis states that the point of tangency of the Mohr failure envelope with the Mohr circle at failure determines the inclination of the failure plane.

Coulomb (1776) was interested in the sliding friction characteristics of different materials and observed that there is a normal stress independent component of the shear strength and a normal stress dependent component. The normal stress dependent component is similar to sliding friction in solid and he termed it as angle of internal friction and denoted by \( \phi \). The other component appears to be related to the intrinsic cohesion of the material and denoted by the symbol \( c \). Coulomb gives the following linear equation for shear strength of soil involving parameters \( \Phi \) and \( c \).

\[
\tau_f = c + \sigma \tan \phi
\]

Eq. (6.3) is presented in terms of total stress, \( \sigma \) on the failure plane. The equation can be rewritten for effective stresses as follows.

\[
\tau_f = c' + u \tan \phi' = c' + \sigma' \tan \phi'
\]

where, \( c' \) and \( \phi' \) are called effective stress parameters as against total stress parameters \( c \) and \( \phi \) respectively. \( u \) is the pore water pressure.
It is always easier to work with straight line relationship and the simpler thing is to do is to straighten out the curved failure envelope of Mohr within the stress range encountered from foundation soil. According to Coulomb it is sufficient to approximate the shear stress on the failure plane as a linear function of the normal stress. Thus was born of Mohr Coulomb failure theory that at failure the stress condition at the rupture surface can be defined by the straight line equation (Eq. 6.3 or 6.4) given by Coulomb touching the circle of stresses given by Mohr. It is a point of concern that if the shear strength parameters \( c \) and \( \varphi \) are to be obtained from failure envelope, it is obvious that the plot of \( \sigma \) and \( \tau \) uses similar scales in both the co-ordinates.

### 6.5 Determination of Shear Strength Parameters \( c \) and \( \varphi \)

Shear strength (its parameters) of soils can be determined both in the laboratory or in the field. The tests for studying the shear strength and related deformation characteristics of soils include,

- **Direct shear test**
- **Triaxial test**
  - Triaxial Compression
  - Triaxial Extension
- **Unconfined compression test**
- **Vane shear test**
- **Torsion shear test**
- **Ring shear test (double shear)**
- **Hollow cylinder test**
- **Plain strain test**
- **Cuboidal or true triaxial test**
- **Borehole shear test**

The first three are the most common laboratory tests. The fourth one, vane shear test, in principle is similar both in the laboratory and field except that the apparatus used in the field is larger in size for practical reasons. The vane shear test is usually done to determine the undrained shear strength parameter (\( c \)) of saturated cohesive soil.

The other tests are mostly used for research purposes and hence are not dealt with here. There are also some indirect ways of measuring the shear strength of soil in the field like Standard Penetration Test (SPT), Cone Penetration Test (CPT) etc.. These tests involve the measurement of resistance of the soil to penetration of a hollow cylindrical split tube or a cone while driven or pushed in to the soil at a desired depth. These procedures are rather empirical in nature, though widely been used throughout the world for their simplicity and inexpensiveness. However, engineering judgement and experience play the most important role to correlate the results which depends on the type of the soil and also on the other factors. The common laboratory tests and their salient features are described below.
**Direct Shear Test**

The direct shear test is a simple and widely used test for determining the shear strength of soil. The direct shear apparatus is essentially a rectangular or circular box having separated lower and upper halves (sections). A schematic diagram of direct shear apparatus is shown in Fig. 6.6. The lower half of the box is fixed to a frame whereas, the upper section is capable of moving horizontally relative to the lower one. The soil sample is placed in the box, with approximately half of the sample within either section. The sample size is usually **60 mm square or 75 mm circular** having a **thickness in between 20 mm to 25 mm**. In case of cohesive soil prism of soil is either prepared or taken from the undisturbed sample using a cutter of similar dimension of shear box. Whereas, for cohesionless soil, the specimen has to be prepared in the box at the required void ratio. Porous disc may be placed on top and bottom of the specimen to facilitate the drainage conditions. A normal load is applied to the plane of shear via a loading plate placed over the sample (Fig. 6.6). The upper half of the shear box is then moved laterally forcing the sample to shear across the plane between the two halves of the box either by controlling rate of strain or stress.

In strain controlled apparatus, the shearing deformation is continuously applied at a constant speed and shear force is measured by means of a proving ring or a load transducer. Such a test can be continued even after the failure of the specimen. **In stress controlled apparatus, the magnitude of the shear stress is increased uniformly or in increments.** In case of load increment, each increment is applied and held constant until the shearing deformation ceases. A stress controlled test cannot be continued as soon as the specimen fails and as such the strain controlled preferred. Not only that a mechanically operated strain controlled apparatus is simplest to devise.

![Diagram of Direct Shear Test](image_url)
Direct shear test
Leveling the top surface of specimen
Preparation of a sand specimen
Specimen preparation completed
Pressure plate

**Fig. 6.6** Direct shear apparatus (a) set up; (b) sample preparation; (c) Schematic diagram.

In direct shear test the test procedure is repeated using at least three identical specimens with variable normal stresses.

It is sometimes convenient to interpret the results, if the normal stresses are so chosen that one is closer to the existing effective overburden and the one of each of the other two on either side of the overburden. Usually, a record of magnitude of the shearing force and the shear displacement is maintained to obtain the peak shear stress (failure stress) from the shear stress-strain plot for a particular normal load. Changes in sample thickness that occurs during the shearing process are also recorded so that the volume change versus shearing stress or shearing strain can be studied.

**Test Results**

Typical shear stress displacement and volume change displacement curves for a single specimen (one normal load) are shown in Fig. 6.7. The slope of the stress strain curve is known as modulus of elasticity or tangent modulus of elasticity of soil, \( E_s \). This modulus varies directly with the stiffness of a soil; the more its value, the more is the stiffness and strength. The peak value of shear stress or the maximum value at a relatively higher shear deformation (normally 20 per cent) gives shear strength of the soil at a particular normal load.
Normal stress = \sigma_3

Normal stress = \sigma_2

Normal stress = \sigma_1

Shear stress at failure, \tau_f

Mohr – Coulomb failure envelope

Shear stress at failure, \tau_f

Overconsolidated clay (c’ ≠ 0)

Normally consolidated clay (c’ = 0)

(a)

(b)

Fig. 6.7 Direct shear test results; (a) Cohesionless soil, (b) Cohesive soil (Drained Test)

A best possible straight line is fitted to the observed point to represent the failure envelope. The shear strength parameters c’ and \phi’ is thus obtained from the plot. It is to be noted that usually we get the drained strength parameters from direct shear test. For granular soil as the cohesion is zero the available strength parameter would be only \phi’.
The effective angle of friction for a particular density is obtained by plotting the maximum value of shear stress $\tau_f$ against the effective normal stress $\sigma'$. At least three tests are carried out at different normal stresses and a straight line passing through the origin and the respective points defines the failure envelope for sand soil (as $c = 0$). The slope of the line gives the value of $\varphi'$. Typical values of $\varphi'$ are shown in Table 6.1.

Table 6.1 Typical values of effective angle of internal friction for granular soil (after Terzaghi and Peck, 1967)

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Effective angle of internal friction, $\varphi'$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loose</td>
</tr>
<tr>
<td>Nonplastic silt</td>
<td>27-30</td>
</tr>
<tr>
<td>Silty sand</td>
<td>27-33</td>
</tr>
<tr>
<td>Uniform sand</td>
<td>28</td>
</tr>
<tr>
<td>Well graded sand</td>
<td>33</td>
</tr>
<tr>
<td>Sandy gravel</td>
<td>35</td>
</tr>
</tbody>
</table>

**Other observations**

The design of normal shear box does not allow control of drainage of the sample. Sands and gravels are free draining materials and they will shear under fully drained conditions. Clay deposits, however, depending on the rate at which the soil mass is stressed, a soil element in the field may fail either completely undrained (no dissipation of excess pore water pressure), partially drained (some dissipation) or fully drained (complete dissipation). Although attempts could be made to measure the undrained strength by shearing a sample rapidly in a few minutes, because there is no control over the drainage there would be a degree of uncertainty as to whether this represented the true value of the undrained strength. For this reason the undrained shear strength of a clay soil is usually measured by the more sophisticated triaxial test.

The direct shear test may, however, be used to measure the drained strength of clay soils by first consolidating the sample fully under the applied normal load and then shearing the sample at a sufficiently slow rate to ensure the full dissipation of pore water pressure generated by the applied shear stress. Hence, for drained clays and sands the effective stress normal to the shear plane is given by $\sigma' = N/A$ and the shear stress $\tau = S/A$, where $N$ and $S$ are applied normal and shear force respectively on the plan area of shear box, $A$.

Typical stress strain volume change relationships for drained tests on loose and dense sand are shown in Fig. 6.8. The shearing in a loose sand rearranges the particles to fill into the voids causing a denser packing and a decrease in volume. At larger strains of (approx. 20%) the sample shears at a constant volume under a constant value of shear stress. At this large strains the tendency for a volume increase as some particles move up and over each other is foiled by adjacent particles moving down into the void space created, resulting in zero net volume change. This condition of soil is known as critical void ratio state. For a dense sand, the interlocking grains have to move apart in order to allow relative movement or shear between the particles to occur. Thus the sample expands during shear and this phenomenon is known as dilatancy. The consequent result is the work against the confining pressure. The peak shear stress therefore occurs at that particular value of shear strain at which dilatancy rate is maximum. With increasing shear strain, the dilatancy rate decreases as the sample approaches to a constant void ratio and shear stress decreases to a residual value. For the same confining pressure, the residual shear stress of a dense sample is equal to the maximum shear
stress of a loose sample. Typical failure strains for loose sands are around 12-16 percent and for dense sands 2-4 percent.

Stress-strain-volume change relations for normally consolidated and over consolidated soils (for drained condition) are very much similar to loose and dense sands respectively. However, the normally consolidated clay will have only friction parameter ($\varphi'$) which in fact dictates its depositional behaviour. Overconsolidated clay test results would be in the form of $c'$ and $\varphi'$. The $\varphi'$ value for clay soil is not influenced appreciably due to over consolidation and generally ranges from about 20° to 30°, decreasing with increasing plasticity index. Overconsolidated clays are characterised by an intercept on $\tau$ axis which is the effective cohesion $c'$. Its value usually ranges from 5 to 30 kN/m$^2$. For both normally consolidated and over consolidated soils there is a decrease in shear stress from its peak to residual, and for the latter it is prominent. The residual angle of friction may be even 9° less than that at peak stress. The stress strain volume change behaviour of various types of soil are shown in Fig. 6.8

![Stress-strain relationship](image)

Fig. 6.8 Stress-strain-volume change relationships for drained tests on clays

**Triaxial Test**

The triaxial test is perhaps the most versatile and sophisticated test for determining the shear strength of soils. Triaxial tests are basically of two types depending on the mode of application of load to represent the field conditions; (i) triaxial compression test and (ii) triaxial extension test. However, it is very unusual where the latter type of test condition prevails. As such, type (i) test is the most conventional one and meets most of the purposes, is briefly described herein.

In the conventional triaxial compression test, a cylindrical soil sample usually 38 mm in diameter and 76 mm in length, is prepared and encased in a thin rubber membrane which extends over a top cap and bottom pedestal. It is then placed in a cylindrical perspex chamber where an all around or confining pressure can be applied. The specimen of other dimensions can also be tested provided the length diameter ratio should lie in between 2:1 to 3:1. To facilitate proper drainage condition of the sample, porous stones/discs and filter papers are used at the edges of the sample. In order to enhance further drainage, filter paper strips are wrapped around the sample. These connect the
porous disc at the top cap, the take off being through a nylon tube which passes out of the cell through its base. A schematic diagram of triaxial test set up is shown in Fig. 6.9.

![Diagram of triaxial test set up](image)

**Fig. 6.9** A schematic diagram of triaxial shear test set up.

The prepared soil sample is properly placed in the cylinder, the cell is the filled with water via cell pressure control tube and subjected to a desired cell pressure. The water pressure, commonly referred to as confining pressure, acts horizontally on the cylindrical surface of the sample through the rubber membrane as well as vertically through the top cap. The pore water pressure in the sample is measured through a porous disc at the base pedestal and connected through a water filled duct to a pressure transducer. To minimise the friction at the top and bottom of the sample and allow an unrestricted lateral deformation during shear, greased rubber discs are placed between the sample and the end caps.

An axial load commonly referred to as deviator load (stress), is the applied and steadily increased until the failure of the specimen occurs. The procedure is repeated for different confining pressures and corresponding deviator loads at least three times using three identical soil specimen. A Mohr’s circle for each combination of confining pressure ($\sigma_3$), and total axial stress ($\sigma_1 +$ deviator stress = $\sigma_1$) is constructed. The tangent to the resulting circles becomes the Mohr envelope as shown in Fig. 6.10.

A representation of stressed specimen is shown in Fig 6.10(a). In Fig 6.10(b), $\sigma_{31}$, $\sigma_{32}$ and $\sigma_{33}$ represents the confining pressure ($\sigma_3$) for specimen 1, 2 and 3 respectively; the corresponding axial stresses are respectively $\sigma_{11}$, $\sigma_{12}$ and $\sigma_{13}$.
Fig. 6.10 Results of triaxial test; (a) Representation of stressed specimen; (b) Mohr’s circle and failure envelope for three different combinations of principal stresses.

In unavoidable circumstances, a smaller number of specimen can be used. Sometimes a single specimen can be loaded to near failure for a particular confining pressure, then the confining pressure is increased to next higher stage and again loaded near to a failure, and then the final cell pressure is applied to loaded it to failure. This type of test is commonly referred to as stage loading. However, in all these circumstances the interpretation of results calls for careful engineering judgement skill.

The lateral pressure produces a normal horizontal stress on all vertical planes of a specimen. Hence any two vertical planes of the specimen experience equal stress $\sigma_3$ (Fig. 6.10.a). No shear stresses are produced on the horizontal planes or any two orthogonal vertical planes by either the axial load or the confining pressure. That is, the vertical and horizontal planes of the element shown are subjected only to maximum and minimum normal stresses $\sigma_1$ and $\sigma_3$ (principal stresses), respectively.

The triaxial test is sophisticated in the sense that it may be manipulated to within a reasonable estimated in situ conditions. The apparatus is designed to allow:

- **Control of confining pressure and deviator stress**
- **Control of porwater pressure and drained water**
- **Saturation of soil sample**

As such **a wide variety of tests are possible in triaxial shear testing** to depict insitu loading and drainage conditions, as shown in Fig. 6.11.
The following tests are adequate for most of the engineering purposes.

- **Unconsolidated Undrained test** (*UU or Quick or Q test*)
- **Consolidated Undrained test** (*CU or Q, or R test*)
- **Consolidated Drained Test** (*CD or Slow or S test*)

(* it is simply because, R lies between Q and S in alphabetic order as is the case of CU with respect to UU and CD*)

Generally, there are three different and successive steps in a triaxial test. These are:

- Preparation of soil sample
- Application of cell pressure
- Application of excess axial stress at constant cell pressure

The first step, preparation of soil sample, for all the soil sample is essentially similar with only a minor exception that in case of *UU* test the drainage accessories like wrapping of filter paper around the soil specimen can be omitted. The dissipation of pore water pressure or drainage during second and third steps of testing depends upon the test type. Often pore water pressure is measures in consolidated undrained test to estimate the effective stress parameters. Such a test is designated as *CU* test. Table 6.2 presents the drainage and loading conditions during second and third steps.

Table 6.2 Drainage and loading conditions in different types of triaxial tests.
<table>
<thead>
<tr>
<th>Type of test</th>
<th>Second Step: Application of cell pressure</th>
<th>Third Step: Application of additional axial stress at a constant cell pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconsolidated Undrained (UU)</td>
<td>Drainage is not allowed. Sample is unconsolidated.</td>
<td>Drainage is not allowed. That is, failure occurs in undrained condition.</td>
</tr>
<tr>
<td>Consolidated Undrained (CU)</td>
<td>Drainage is allowed. That is, sample is consolidated.</td>
<td>Drainage is not allowed. That is, failure occurs in undrained condition.</td>
</tr>
<tr>
<td>Consolidated Drained (CD)</td>
<td>Drainage is allowed. That is, sample is consolidated.</td>
<td>Drainage is allowed. That is, failure occurs in drained condition.</td>
</tr>
</tbody>
</table>

**Shear strength parameters obtained from various types of triaxial test**

The triaxial test data obtained from a test on each of the specimen are separately plotted as deviator stress strain diagram to obtain the peak or maximum deviator stress and the corresponding axial strain. The excess pore water pressure reading at the failure strain is used to calculate the effective principal stresses where appropriate. Thus the principal stresses on the soil sample are estimated and Mohr circle are drawn. For details of the test procedure and computations reference can be made to Lambe(1993).

**Shear strength parameters for cohesive soil**

The shear strength of cohesive soils depends on many factors like degree of saturation, stress history, loading rate and drainage conditions. Cohesive soils are often treated in a saturated state to represent the worst condition that can prevail in the field, though the parameters estimated will be generally on the conservative side. Even then, researchers, concentrated many of their studies on unsaturated and remoulded clays to assess the insitu behaviour by analysing the results obtained from their studies.

**Unconsolidated undrained (UU) test**

In this test a minimum of three soil samples are subjected to different confining pressure $\sigma_3$ and then loaded to failure. Before applying this confining pressure, a small confining pressure and equal amount of pore water pressure may be applied to the soil and allowed to stay for some time to saturate the sample. This application of pore water pressure to the sample is known as back pressure. The degree of saturation of the soil specimen can be checked from the pore pressure parameter, $B$ as discussed later.

When a saturated cohesive soil is subjected to a stress increase, it will respond simply by increasing the pore water pressure by that amount. So, any change in all around pressure or axial stress will not bring any change in the structure or arrangement of soil particles within the soil mass. Hence the undrained strength remains constant and independent of cell pressure. That is, the soil will fail at an equal deviator stress for all the different cell pressures. Deviator stress which is the diameter of the Mohr’s circle being equal, the failure envelope will be a horizontal line thus depicting that only cohesion parameter will contribute to generate the shear strength of soil. The cohesion parameter at
this condition is termed as undrained cohesion and designated as $c_u$. The other parameter $\phi$ would be zero. A typical example of the Mohr's circle, failure envelope and the estimated shear parameter is shown in Fig. 6.12. The failure envelope in this case can be defined as

$$\tau_f = c_u$$  \hspace{1cm} (6.5)

![Mohr's Circle Diagram](image)

Fig. 6.12 Unconsolidated undrained (UU) strength of saturated clay

**Consolidated undrained (CU) test**

In this test soil sample is initially consolidated under confining pressure. This pressure is known as initial effective confining pressure. The volume change reading can also be taken at this stage to estimate the initial void ratio of the specimen. After consolidation, the confining pressure and back pressure may be increased by equal amount thus keeping the effective cell (confining) pressure constant. As usual, the back pressure is applied to saturate the soil sample and also to facilitate the measurement of negative pore water pressure during application of deviator stress in case of an overconsolidated clay. The soil sample is sheared at this elevated cell pressure ($\sigma_3$) with a deviator stress of, that is axial stress of $\sigma_i(=\sigma_3+\Delta\sigma)$. Morh’s circles are drawn for the three specimens to determine the failure envelope and strength parameters in terms of total stress. The Mohr’s circle diagram and failure envelope will be similar to the generalised diagram of Fig. 6.10b as reproduced in Fig. 6.13. The strength parameters would be designated as $c_{cu}$ and $\phi_{cu}$ instead of $c$ and $\phi$ as mentioned there. The failure envelope will be represented by

$$\tau_f = c_{cu} + \sigma \tan \phi_{cu}$$ \hspace{1cm} (6.6)

As mentioned earlier, consolidated undrained test can also be carried out with pore water pressure measurement while shearing the soil sample. The effective stress parameters in such a case are determined drawing Mohr circle and hence the failure envelope taking in to consideration the effective cell pressure $\sigma'i$ as ($\sigma - u_i$) and effective axial stress $\sigma"i$ as ($\sigma - u_i$). Here $u_i$ is the pore water pressure at failure. Thus the effective minor and major principal stresses $\sigma'i$ and $\sigma"i$ respectively are to be computed for all the three specimens.
Fig. 6.13 Consolidated undrained (UU) strength of saturated clay
It is understandable that the Mohr circles for this effective stress condition would be of similar diameter as those for total stress condition. Only thing is that they will shift towards left in the diagram thus resulting in a higher \( \varphi \) and lower \( c \) values in terms of effective stress. A typical illustration is shown in Fig 6.14.

Fig. 6.14 Total and effective stress parameters from a CU test (Typical one circle of each are shown, others are hidden)

The effective strength parameters are \( \bar{c}_{cu} \) and \( \bar{\varphi}_{cu} \) respectively for cohesion and friction and the equation for failure envelope is

\[
\tau_f = \bar{c}_{cu} + \sigma' \tan \bar{\varphi}_{cu} \tag{6.7}
\]

However, for normally consolidated soil \( \bar{c}_{cu} \) would be zero as explained later.
**Consolidated drained (CD) test**

In this type of test, the sample is first consolidated under confining or cell pressure. Back pressure may be applied in a similar manner to that of *CU* test to saturate the sample, if necessary. Thus the confining pressure $\sigma'$ here is also the effective confining pressure $\sigma''$ as that of *CU* test. The specimen is then sheared to failure with a very small strain rate so that no excess pore water pressure can develop on the failure surface. That is, the pore water pressures remains zero at all the stages of the test thus prevailing effective stress conditions allthrough. The volume change reading can also be taken to examine the void ratio of the soil at failure condition. Mohr's circle interms of effective minor and major principal stresses at failure for all the three specimen are are drawn to obtain the effective shear strength parameters $c'$ and $\phi'$. The failure envelope is of typical in nature as represented by Fig. 6.10b, and $c$ and $\phi$ would be replaced by the parameters $c'$ and $\phi'$. Fig. 6.15 shows the related diagrams. The values of these parameter are different. The failure envelope can be given by:

$$\tau_f = c' + \sigma' \tan \phi'$$  \hspace{1cm} (6.8)

The parameters $c'$ and $\phi'$ are often replaced by $c_{CD}$ and $\phi_{CD}$ to remind that these parameters are for drained condition. The parameters obtained from a drained test always refers to effective stress conditions though these can also be obtained from a *CU* test and that should be specifically spelled out. However, in laboratory testing there may be slight differences in effective stress parameters determined from *CU* test and CD tests. However, the magnitude strength obtained from a CD test is always higher than that form a *CU* test for a normally consolidated clay.

![Failure envelope](image)

Fig. 6.15 Shear strength parameters from a consolidated drained (CD) test.

**Strength parameter for cohesionless soil**

When load is applied to a soil, the soil is stressed and causes the soil-water matrix to compress and some of the water is squeezed out. Because cohesionless soils have a high hydraulic conductivity (coefficient of permeability), this water is able to move quickly and easily. The potential drainage rate is atleast as high as the loading rate. This condition is known as drained condition and the stresses acting on a cohesionless soil are always the effective stresses. Hence, in case of cohesionless soils the measured strength parameters from shear tests are always the effective one and it is almost
independent of type of triaxial tests. As cohesionless soil has smaller surface forces, the cohesion parameter can be considered as zero the the strength envelope can be explained by the equation

$$\tau_f = \sigma' \tan \varphi'$$  \hspace{1cm} (6.9)

As such, only one Mohr’s circle is required to define the straight line failure envelope of strength as another point is known to be the origin of $\sigma - \tau$ coordinate system. However, confining pressure has a little effect on strength the parameter and as such, conventionally three specimen are tested at different confining pressures, Mohr’s circles are drawn to obtain the failure envelope of the form represented by Eq. (6.9) and thus the angle of internal friction $\varphi'$ is determined. A typical Mohr circle diagram for triaxial test on dry cohesionless soil is shown in Fig. 6.16, where the strength parameter $c$ is zero.

![Mohr's circle diagram](image)

Fig. 6.16 Shear strength results of cohesionless soil from a triaxial test

However, the moist cohesionless soil might show a very small value of apparent cohesion which is due to surface tension. In all the shear strength test of sand volume change is a very important factor. Hence, it is preferable to record the volume change readings for better explanation of the test results.

*It is to be noted that due to the mode of formation of failure planes, the shear strength parameter $\varphi'$ obtained from direct shear test overestimates the triaxial $\varphi'$ value approximately by 10%.*

**Strength of normally consolidated and over consolidated clays**

Clay may exist in the field under two conditions;

- **existing effective overburden pressure**, $\sigma'_o$ is greater than or equal to the maximum of effective pressures subjected in the past called overconsolidation pressure $\sigma'_o$. This type of soil is known as normally consolidated soil.

- **existing effective overburden pressure** ($\sigma'_o$) is less than the maximum past pressure($\sigma'_o$). This type is over consolidated soil.

The ratio $\sigma'_o/\sigma'_o$ is known as over consolidation ratio (OCR). Strength characteristics of normally consolidated and over consolidated clays (saturated) differ because of the fact that their depositional modes and drainage behaviour are responsible to impart the cohesion and frictional properties between the particles. As such the strength characteristics of clay soils are dependent on $OCR$. 

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The UU test on both normally consolidated and overconsolidated clays gives \( c \) parameter that is shear strength in this case equals to \( c \) only. However, the strength for a overconsolidated clay is higher than the similar clay in a normally consolidated state. The difference may be attributed to the increased density and the negative pore pressures in over consolidated states. The strength measured in a \( CU \) test comes from both \( c \) and \( \varphi \) parameters in each of the caes. While compared the strength, it depends on the over burden pressure and \( OCR \).

For a \( CU \) test with pore pressure measurement or \( CD \) test , normally consolidated soil gives \( \varphi' \) (drained \( \varphi \)) parameter resulting in a zero cohesion intercept. However, for over consolidated soils we get both \( c' \) and \( \varphi' \). Strictly speaking, the failure envelopes for overconsolidated clays are nonlinear. For a over consolidated soil, if the failure envelope of the normally consolidated range is extended backwards it will pass through the origin giving \( c' \approx 0 \).

Undrained shear strength of normally consolidated clays varies with effective overburden pressure (i.e. with depth). Several empirical relations are available. Of them, the one given by Skempton is mentioned below.

\[
\frac{c_u}{\sigma'_o} = 0.11 + 0.0037I_p \tag{6.10}
\]

Where, \( c_u \) is the undrained cohesion, \( \sigma'_o \) is the effective over burden pressure and \( I_p \) is the plasticity index in percent. Eq. 6.10 afford a quick but approximate estimation of the increase in undrained shear strength of a normally consolidated clay with depth.

**Unconfined Compression Test**

Unconfined compression test is essentially a special case of triaxial test where the minor principal stress or cell pressure is kept as zero. No rubber membrane is required to encase the soil sample As such, the apparatus becomes much simpler. This is the easiest, quickest and simplest test for determining the undrained strength of saturated cohesive soil. This test can also be done on unsaturated sample, but it should be borne in mind that undrained shear strength of cohesive soil varies significantly with the moisture content and density and in the design due consideration should be given. Similar to triaxial tests, three cylindrical specimens are tested for axial compression to failure in an unconfined compression machine and their average values are taken as the strength.

The unconfined compression testing machine essentially consists of a loading frame where the cylindrical soil sample can be placed in between base platform and a top cap. The top cap is attached to either with a proving ring or load transducer. The bottom platform can be raised either by gear system or by a hydraulic pump system. A view of the machine is shown in Fig. 6.17 and the failure envelope is shown in Fig. 6.18.
Theoretically, an unconfined compression test should yield the same results of undrained shear strength as obtained from triaxial UU test. However, this supposition is only valid if several assumptions are satisfied.

- The soil specimen must be clay and 100 percent saturated.
- The specimen must not contain any fissures, cracks, silt seams, verves or any other defects.
- The specimen must be sheared rapidly to failure. If the time is too long, evaporation and surface drying may result in high strength. Typical time to failure is 5 to 15 minutes. If the sample does not show any sign of failure, the unconfined compressive value is taken as the as 20 percent strain.

Unconfined compressive strength also gives a measure of consistency of clay soil as expressed in Table 6.3. Unconfined compression test can be done both on undisturbed and remoulded samples. Their values usually gives some useful estimation about the sensitivity of clay soils.
Table 6.3  General relationship of consistency and unconfined compressive strength of clay soil.

<table>
<thead>
<tr>
<th>Consistency</th>
<th>Unconfined compression strength, $q_u$(kN/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very soft</td>
<td>&lt; 25</td>
</tr>
<tr>
<td>Soft</td>
<td>25 - 50</td>
</tr>
<tr>
<td>Medium</td>
<td>50 - 100</td>
</tr>
<tr>
<td>Stiff</td>
<td>100 - 200</td>
</tr>
<tr>
<td>Very stiff</td>
<td>200 - 400</td>
</tr>
<tr>
<td>Hard</td>
<td>400 - 800</td>
</tr>
</tbody>
</table>

Sensitivity describes the effect of stress and environment with time on the strength of the soil. It is usually defined as the ratio of undrained shear strength of clay soil in undisturbed condition to that at a remoulded state at the same water content. Though, it is referred to undrained strength it is customary to use the unconfined compressive strength to define its sensitivity. Hence,

$$Sensitivity, S_t = \frac{q_u\text{ (undisturbed)}}{q_u\text{ (remoulded)}}$$

The sensitivity of a soil measures the reduction of shear strength that might occur due to disturbance of the sample. Bjerrum (1954) gives a sensitivity classification of soil which is presented in Table 6.4.

Highly overconsolidated soils are insensitive. This is partly due to low natural water content in the soil deposit. Glacial clays lie in the range of medium to extra sensitive classification. Some of fresh water and marine deposits are quick. However, sensitivity most of the clay deposits ranges from 2 to 4. The disturbance or the decrease in strength of clay soil is attributed at least partly due to disturbance of adsorbed water in clay layers. If the remoulded soil is left undisturbed at the same water content, partly of its strength is regained due to reorientation of clay particles. This is known as thixotropy.

Table 6.4  Sensitivity Classification of Clay Soil

<table>
<thead>
<tr>
<th>Sensitivity Value</th>
<th>Soil Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 2</td>
<td>Insensitive</td>
</tr>
<tr>
<td>2 - 4</td>
<td>Medium sensitive</td>
</tr>
<tr>
<td>4 - 8</td>
<td>Sensitive</td>
</tr>
<tr>
<td>8 - 16</td>
<td>Very sensitive or Extra Sensitive</td>
</tr>
<tr>
<td>16 - 32</td>
<td>Slightly quick</td>
</tr>
<tr>
<td>32 - 64</td>
<td>Medium quick</td>
</tr>
<tr>
<td>Greater than 64</td>
<td>quick</td>
</tr>
</tbody>
</table>
Vane Shear Test

The vane shear test can be applied for measurement of undrained shear strength of clay soil both in the laboratory and field. In particular this test is very suitable for soft clays, which otherwise may be greatly disturbed during the extraction and testing process.

Essentially, the vane shear apparatus consists of a series of thin rectangular blades welded to a circular shaft, so that when the shaft is rotated about its axis each blade generates the same cylinder. In its usual form, four blades are used forming a cross section. A schematic diagram of vane shear apparatus is shown in Fig. 6.17(a).

![Diagram of vane shear apparatus](image)

**Fig. 6.17** Vane shear test; (a) diagram of apparatus; (b) distribution of stresses

The vane dimensions may vary but a height/diameter ratio of 2.0 is commonly used. The vane is pushed into the soil to a predetermined depth and rotated rapidly (usually 0.1°/sec) until shear takes place over the surface area approximately equal to that generated by the vane. The strength is estimated from the maximum measured torque required to shear the cylindrical surface. A simple formula between shear strength and torque can be derived depending on the following assumptions.

- The soil is purely cohesive \((\varphi = 0)\), homogeneous and isotropic.
- The insertion of the blade causes no disturbance to the soil.
- No drainage takes place during shearing.
- Failure takes place by shearing over the surface of the cylinder generated by the rotating vane.
- No progressive failure takes place in the soil and the shear strength is fully mobilised on the surface of the cylinder at failure.

The total torque is the sum of the opposing moments on the cylinder side and on the two edges of the cylinder. Mathematically, it can be written as:

\[
T = M_{\text{Side}} + 2 \times M_{\text{Edge}}
\]
Where,

\[ T = \text{total applied torque} \]
\[ M_{side} = \text{resisting moment due to the shear forces on the cylindrical surface} \]
\[ M_{edge} = \text{resisting moment due to shear forces on a single edge surface}. \]

Let us consider a cylinder of soil of height, \( h \) and diameter, \( d \) (Fig. 6.19) will resist the torque until the soil fails. If the assumption of full shear mobilisation at the edges are not valid, there could be various assumptions regarding the distribution of mobilised shear strength at the edges. The possible distributions are shown in Fig. 6.19(b). Accordingly, the expression for resisting torque can be obtained by integrating the resulting moment for the relevant stress distribution diagram of the edges and adding the value for the moment due to surface resistance. The general expression can be obtained as follows:

\[
T = \pi c_u \left( \frac{a^2 h}{2} + \frac{ad^3}{4} \right) \quad (6.13)
\]

where,

\[ c_u = \text{undrained shear strength of soil} \]
\[ a = \begin{cases} 
\frac{2}{3} & \text{for uniform distribution of shear at the edges (fully mobilised strength)}, \\
\frac{1}{2} & \text{for triangular distribution of shear at the edges}, \\
\frac{3}{5} & \text{for parabolic distribution of shear at the edges}. 
\end{cases} \]

Sometimes, a vane shear test is done in a borehole where the top edge of the vane coincides with the bottom of the borehole. In such a case only one edge instead of two partakes in the resistance process. The value of ‘a’ in Eq. (6.13) will then be reduced to half of the values as given above. For a conventional event, where a uniform distribution at both the edges is considered and the height of the vane is twice the diameter of the failure cylinder, Eq. (6.13) takes a simplified form to estimate the undrained cohesion of soil as:

\[
c_u = \frac{6T}{7\pi d^3} \quad (6.14)
\]

6.6 Pore Water Pressure Parameters

Pore water pressure plays a significant role in determining the shear strength of soil. **The change in pore water pressure due to change in applied stress is characterised by dimensionless coefficients called ‘pore pressure coefficients’ or ‘pore pressure parameters A and B’**. These parameters have been proposed by Skempton(1954). In undrained triaxial compression test, pore water pressure develops during the application of cell pressure and also during the application of additional axial stress.

The triaxial test, in addition to determining the strength parameters of soil, also gives data for predicting the excess the excess pore water build up at a point in a soil mass by a change in total stress condition.

The ratio of pore water pressure developed due to application of confining or cell pressure is termed as B parameter: Therefore,
\[ B = \frac{\Delta u_c}{\Delta \sigma_c} = \frac{\Delta u_c}{\Delta \sigma_3} \]  
(6.15a)

That is,

\[ \Delta u_c = B \Delta \sigma_3 \]  
(6.15b)

where,

\[ \Delta \sigma_c = \text{increase in confining pressure} \]
\[ \Delta u_c = \text{increase in pore pressure due to } \Delta \sigma_c \]
\[ \Delta \sigma_3 = \text{increase in minor principal stress}. \]

In undrained condition, the decrease of volume of mass is equal to that in the volume of pores. Using the principle of theory of elasticity it can be shown that

\[ B = \frac{1}{1 + \frac{nC_{vo}}{C_{sk}}} \]  
(6.16)

where,

\[ n = \text{porosity of soil} \]
\[ C_{vo} = \text{volume compressibility (change in volume per unit volume per unit pressure increase) of the voids} \]
\[ C_{sk} = \text{volume compressibility of soil skeleton}. \]

For details of derivation reference can be made to Holtz and Kovacs(1981).

In case of a saturated soil, the volume compressibility of void \((C_{vo})\) equals to volume compressibility of water, \(C_w\). The compressibility of water as compared to that of soil skeleton is very small and as such \(\frac{C_{vo}}{C_{sk}}\) can be considered as zero for all practical purposes. Hence, for a saturated soil mass, according to Eq. (6.15), the parameter \(B\) equals to unity.

On the contrary, for a dry soil \(C_{vo}\) takes the value of volume compressibility of air, \(C_a\) which is much greater than \(C_{sk}\). Hence, \(\frac{C_{vo}}{C_{sk}}\) becomes to infinity resulting in a zero value of \(B\). The degree of saturation dictates the value of \(B\) in the other instances. The variation of experimentally determined values of \(B\) with respect to degree of saturation are presented in Fig. 6.18.

![Graph of B vs. S_r](image)

Fig. 6.18 Variation of pore pressure parameter \(B\) with degree of saturation, \(S_r\) of clay soil (after Venkatramaiah, 1995)
The value of B depends also on soil type, stress level and stress history. The exact relationship for a particular soil will have to be determined experimentally. Pore water pressure developed during the application of deviator stress in a triaxial machine can be related as

\[
\overline{A} = \frac{\Delta u_d}{\Delta \sigma_i - \Delta \sigma_3}
\]

That is,

\[
\Delta u_d = \overline{A}(\Delta \sigma_i - \Delta \sigma_3)
\]

(2.17)

where, \( \Delta u_d \) = increase in pore water pressure due to increase in deviator stress (\( \Delta \sigma_i - \Delta \sigma_3 \)).

\( \overline{A} \) is a dimensionless experimentally determined parameter which is the product of another parameter, \( A \) and \( B \). If now \( \Delta u \) denotes the total excess pore water pressure within the soil mass due to increase in confining pressure and deviator stress, then using the principle of superposition, it can be related as:

\[
\Delta u = \Delta u_c + \Delta u_d = B \Delta u_e + \overline{A}(\Delta \sigma_i - \Delta \sigma_3)
\]

\[
= B \Delta u_e + AB(\Delta \sigma_i - \Delta \sigma_3)
\]

That is,

\[
\Delta u = B[\Delta u_e + A (\Delta \sigma_i - \Delta \sigma_3)]
\]

(6.18)

For a 100% saturated sample, where \( B = 1 \) the equation simplifies to the form:

\[
\Delta u = \Delta u_e + A (\Delta \sigma_i - \Delta \sigma_3)
\]

(6.19)

Similar to parameter \( B \), the parameter \( A \) is not also constant. It varies with the soil type, stress level, soil anistropy, density index and stress history (OCR). Bishop and Hankel(1962) gave a relationship between over consolidation ratio and \( A \) value at failure, \( A_f \). This is shown in Fig. 6.19. Typical \( A_f \) values for different clay soils are also given by Skempton. They are presented in Table 6.5.

Skempton’s pore pressure coefficients \( A \) and \( B \) are the most useful parameters in engineering practice in judging the shear strength of soil since they can be used to predict the induced pore pressure whenever the changes in the total stress are known. For example, if the pore pressure response during embankment construction on a very soft clay foundation (undrained loading) is to be estimated, these parameters can be used. The increase in pore water pressure results in the indication of instability. The factor of safety thus can be calculated at a particular stage of loading.

![Fig. 6.19 Variation of \( A_f \) with over consolidation ratio (OCR)](image)

Table 6.5 Values of \( A_f \) for various clay soils.

- 28 -
<table>
<thead>
<tr>
<th>Type of soil</th>
<th>Value of $A_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highly sensitive clay</td>
<td>0.75 to 1.50</td>
</tr>
<tr>
<td>Normally consolidated clay</td>
<td>0.50 to 1.00</td>
</tr>
<tr>
<td>Compacted sandy clay</td>
<td>0.25 to 0.75</td>
</tr>
<tr>
<td>Lightly overconsolidated clay</td>
<td>0.00 to 0.50</td>
</tr>
<tr>
<td>Compacted gravelly clay</td>
<td>-0.25 to 0.25</td>
</tr>
<tr>
<td>Heavily overconsolidated clay</td>
<td>-0.50 to 0.00</td>
</tr>
</tbody>
</table>

**Measurement of pore pressure parameters**

The pore pressure parameters $A$ and $B$ may be determined in a consolidated undrained test. The coefficient $B$ is determined during consolidation stage, and $A$ during the shear stage of the test.

All around pressure (cell pressure) is applied in the sample and drainage valve is kept closed. The change in pore pressure $\Delta u_c$ is measured against the applied confining pressure $\Delta \sigma_3$. The parameter $B$ is obtained as $B = \frac{\Delta u_c}{\Delta \sigma_3}$.

The drainage path is then opened and the sample is allowed to consolidate at the same confining pressure. Once consolidated, the drainage valve is closed and a deviator stress $(\Delta \sigma_1-\Delta \sigma_3)$ is applied keeping the cell pressure constant. The pore water pressure change, $\Delta u_d$ at this stage is recorded.

The parameter is obtained as $A = \frac{\Delta u_d}{\Delta \sigma_1 - \Delta \sigma_3}$ hence $A$ as $\frac{A}{B}$.

### 6.6 Shear Strength of Unsaturated Soil

**Shear strength of partially saturated soils**

In the previous sections, we were discussing the shear strength of saturated soils. However, in most of the cases, we will encounter unsaturated soils.

Pore water pressure can be negative in unsaturated soils.
Bishop (1959) proposed shear strength equation for unsaturated soils as follows

\[ \tau_f = c' + (\sigma_n - u_a) + \chi(u_a - u_w) \tan \phi' \]

Where,
- \( \sigma_n - u_a \) = Net normal stress
- \( u_a - u_w \) = Matric suction
- \( \chi \) = a parameter depending on the degree of saturation
  (\( \chi = 1 \) for fully saturated soils and 0 for dry soils)

Fredlund et al (1978) modified the above relationship as follows

\[ \tau_f = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \]

Where,
- \( \tan \phi^b \) = Rate of increase of shear strength with matric suction

Therefore, strength of unsaturated soils is much higher than the strength of saturated soils due to matric suction.

\[ (u_a - u_w)_2 > (u_a - u_w)_1 \]
\[ (u_a - u_w)_1 > 0 \]
\[ u_a - u_w = 0 \]
Factors Affecting Angle of Shearing Resistance

The shear resistance of sands and gravels is made up of sliding and rolling friction plus the resistance to volume change by interlocking. Interlocking effect is important for dense sands and is the main contributing factor for the increased angle of shearing resistance in them. However, the angle of shearing resistance corresponding to the ultimate strength is found to be not much different for the dense sand and the loose sand. The angle of shearing resistance decreases rapidly with increase in percentage of fine. A fine content of about 20 to 25 per cent may reduce the φ' value for clean sand by 5° to 7°. Table 6.5 shows the typical values for φ'.

Table 6.5 Approximate values of φ' for sand

<table>
<thead>
<tr>
<th>Type of sand</th>
<th>Round grains, uniform grain size distribution</th>
<th>Angular grains, well graded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose sand</td>
<td>28.5°</td>
<td>34°</td>
</tr>
<tr>
<td>Dense sand</td>
<td>35°</td>
<td>46°</td>
</tr>
</tbody>
</table>

A rather elegant empirical formula suggested by Brinch Hansen and Lundgren (1960) takes into account the influence of the major factors in determining φ' in sands and gravels. It is expressed in the form:

$$ \varphi' = 36^\circ + \varphi_1' + \varphi_2' + \varphi_3' + \varphi_4' $$

(6.20)

In Eq. 6.20, \( \varphi' = 36^\circ \) represents the φ' value for average conditions while \( \varphi_1', \varphi_2', \varphi_3' \) and \( \varphi_4' \) account for influence of grain shape, size, gradation and density index respectively.

Grain shape factor, \( \varphi_1' \), is taken on the following basis:
Angular grain = + 1°
Subangular grain = 0°
Rounded grain = - 3°
Well-rounded grain = - 6°

Grain size factor, $\varphi'_2$
Sand = 0°
Fine gravel = + 1°
Medium and coarse gravel = + 2°

Gradation factor, $\varphi'_3$
Poorly graded soil = - 3°
Medium uniformity = 0°
Well graded soil = + 3°

Relative density factor, $\varphi'_4$
Loosest packing = - 6°
Medium density = 0°
Densest packing = - 6°

The influence of relative density is the most important, as can be seen above. Values of $\varphi'$ can range from 20° to 48°.